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ON MATHEMATICS EDUCATION AND CULTURE: CRITICAL AFTERTHOUGHTS

ABSTRACT. This paper centers on the May 1988 Special Issue of *Educational Studies in Mathematics* devoted to mathematics education and culture, which is now available in book form. It questions some of the current tenets about the meaning and significance of the concept "culture" for a theory of mathematics education. More generally, it also argues against the cooping up of scientific problems by dividing the mathematics education community into small circles of experts which behave like a peculiar breed of mutual benefit societies, without giving due attention to the needs of scientific democracy and the simple pursuit of truth. Ultimately, it calls for an open scientific debate, unfettered by moral and ideological prejudices and, whenever necessary, disrespectful of fashionable notions. In this essay, all these points are tackled in close relation to a thorough – and unusually long – review of the book.

CULTURE: A CONCEPT ON PROBATION

The May 1988 special issue of *Educational Studies in Mathematics* on mathematics education and culture has been reprinted in book form.[1] Save for the cover, it is an unaltered edition: even the numbering of the pages has stood the test of republication.[2] All papers are in English. The opportunity could have been seized to remove the misprints, typing errors and slips of the pen that sometimes impede the reader's progress. However, the different contributions that made up the May 1988 special issue are thus readily available to a wider readership.

The topic is an exciting one. The book is thoughtful, richly informative, and intellectually challenging. It left me, however, with mixed feelings. The general argument seemed to me not altogether convincing and, in a certain sense, misleading.

In his presentation (pp. 115-116), editor Alan Bishop – whose work in the field is well-known – briefly introduces the reader to the facts of the case. Interest in the socio-cultural aspects of mathematics education has been steadily gaining ground for the past fifteen years or so. This growing concern among mathematics educators is linked with the aftermath of colonisation and the "enforcement" of Western-type ways of life – a world-wide process that has brought peoples and nations into contact with Western technology and culture, including mathematics education. Not surprisingly, therefore, the first three papers tackle the cases of the Australian Aborigines (Beth Graham), of the now independent peoples of Mozambique (Paulus Gerdes), and of black students in South Africa (Norma Presmeg).

Although it provides a wealth of information, the approach taken here is not free from *theoretical* ambiguity. The concept of culture runs the risk of being applied preferably to those situations with a definite *exotic* ring – be it the exoticism of country, class or sex. Such a subjective perception and interest could no doubt be given objective meaning, but two major considerations should be kept in mind. Firstly, the situations thus examined – including classroom situations – are by no means accountable in terms of *cultural* factors alone. What makes for the distance between situation and observer is, more often than not, sheer violence, imposed by history on those being observed. Two centuries ago, in his unfinished *Ideas on the Philosophy of the History of Mankind* (1784-1791), Herder wrote bluntly: "Men of all

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[1] Alan J. Bishop (ed.), *Mathematics Education and Culture*, reprinted from Educational Studies in Mathematics, Volume 19, No. 2, Kluwer Academic Publishers, Dordrecht/Boston/ London, 1988. ISBN 90-277-2802-X 172 pp., Dfl. 90.

[2] Oddly enough, the book begins on page 117.

quarters of the globe, who have perished over the ages, you have not lived solely to manure the earth with your ashes, so that at the end of time your posterity should be made happy by European culture. The very thought of a superior European culture is a blatant insult to the majesty of Nature".[3] "Cultural sentimentality" may sometimes border on political hypocrisy and lead to scientific fallacy.

The cultural approach tends to focus on symptoms, and easily ignores the root of the evil. Moreover, attraction to obvious conflict situations – which are predominantly situations of *social* and *political*, not only cultural, conflict – may also misdirect investigations. The researcher is tempted to ignore more familiar situations characterised by phenomena truly traceable to cultural causes. *Culture* conflicts and *culture* shocks – let us say, in the face of mathematics and mathematics education – may and do arise *even in the case of Western societies*, even in the case of pupils from the *upper classes* of Western societies. They are not, as such, a privilege of the slums and ghettos of former colonies or working-class districts. The iron law of scientific explanation conflicts here with "epistemological opportunism". No scientific concept can be used in only those cases which please the researcher, and be discarded arbitrarily when he does not choose to use it. The laws of gravity, which apply to falling bodies, also apply to balloons. Rejection of *ad hoc* explanations and theorisations is a major principle of scientific activity. In my opinion, therefore, the concept of culture as we find it here is not yet a fully-fledged concept of the theory of mathematics education; it still carries deep-rooted ideological attitudes. It is, as C. Wright Mills once remarked, a "spongy" concept, whose epistemological status should be clarified.[4]

THE TRADITIONAL APPROACH TO SCHOOLING

The question now arises of how the concept is *used*, of the position that it occupies, of the *function* that it serves as a theoretical tool. Implicit in most contributions to the volume – with very few exceptions to be mentioned in due course – is a certain paradigm, a "working model" whose main characteristics reduce to the following: 1. mathematics as an activity, if not as a body of knowledge, is not *culture-free;* 2. the learner's *cultural* equipment may prove at variance with the cultural prerequisites of mathematical activity and mathematics learning; 3. whenever this is the case, specific learning difficulties follow.

One may choose to ignore those difficulties, considering them as the inescapable lot of whoever is introduced to a new domain of knowledge or social practice. More than that, the culture conflict which is then expected to arise may be granted positive value, as a token of an ongoing process of *acculturation*, seen itself as a *legitimate change for the better*. This is the traditional view of education: education as enlightenment, designed to "put off the old man", not only through learning but also by means of an overall change in values, norms and attitudes.

The paper by Marc Swadener and R. Soedjadi on the treatment of values in the Indonesian educational system (pp. 193-208) follows this traditional pattern. The authors briefly discuss the question of values in education and aptly contrast the condition of (American) public schools, where the problem of values is always more or less critical, with that of private schools, in which "values teaching" is generally in tune with the views espoused by sponsoring organisations. However the body of the discussion deals with the Indonesian case. Public education in the Republic of Indonesia revolves around a founding document called

[3] This quotation is borrowed from Raymond Williams' excellent book, *Keywords*, "a vocabulary of culture and society" (Williams, 1983).

[4] "In contrast with social structure, Mills observes, the concept 'culture' is one of the spongiest words in social science, although, perhaps for that reason, in the hands of an expert, enormously useful. In practice, the conception of 'culture' is more often a loose reference to social milieux plus 'tradition' than an adequate idea of social structure (Mills, 1959, p. 160).

the *Panca Sila* – the “Five Principles” – which concisely define the basic attitudes required of citizens. Along with mathematics, language, science, religion, etc., every Indonesian school offers courses in “Panca Sila ethics”, but all teachers are called on to participate in a nation-wide scheme to “develop pupils’ personalities” in consonance with the Five Principles.

In this line of thought, the authors essentially try to show how mathematics education can contribute its share to the nation’s united efforts. The undertaking seems at first promising, but its outcome is rather disappointing (to say nothing of the treatment inflicted on “Freud’s psychoanalytic theory of personality”, for instance). The core of the argument consists of a number of examples drawn from mathematical practice, and supposed to echo major values of the new ethics. The first-listed value is “Universe”: solutions to mathematical problems generally depend on which universe – let us say, which number system, or which metric, for instance – one considers. Typical of the authors’ manner is the following statement: “… examining the concept of ‘universe’ further develops the student’s consciousness of limitations, i.e. the limitations of the environment in which the problem is considered, *and ultimately the society in which the student exists.* Such consciousness can reduce tension. This implies an educational and philosophical value that reducing tension is desirable” (p. 202; *italics added).* Unfortunately, we are kept in the dark as to how such mathematical idiosyncrasies – which truly belong to mathematical experience and (sub)culture – are supposed to be transferred to other societal contexts of the pupil’s experience.

While the reasoning behind the argument remains obscure, the paper offers a clear view of the real stakes. Such a study pertains to the category of what I would call *apologetical discourses.* To put it plainly: mathematics “noospherians” – members of the noosphere (reviewer’s personal jargon), i.e. members of the mathematics education intelligentsia – have “to put over their goods”, to convince society that mathematics and, therefore, mathematics education, are highly *beneficial* to society. However subtly, fighting in defence of mathematics and mathematics teaching is the common lot of most of the literature on mathematics education, and the other contributions to the volume are no exceptions to this rule.

A CONTEMPORARY DAYDREAM

In the recent decades, however, attacks on mathematics education have led most noospherians to rely on a more flexible strategy. Culture contlicts in classroom situations, which used to be seen as inevitable and even beneficial, are now increasingly regarded as destructive of the learner’s culture. Mathematics, one has come to realise, is thus neither culture-free nor culture-fair. This is where the other papers part company with the “traditional”, now almost forgotten, approach to apologetics. The general idea behind the new approach is to transform the conditions under which the encounter of pupil with mathematics takes place. This implies a radical change that is increasingly advocated in noospheric circles and consists in what might be termed the “enculturation” of mathematics. It is, I would observe, a desperate attempt to prove that mathematics is *not foreign* to the child’s everyday experience. This is where the newly-introduced concept, ethnomathematics, is usually called on. From such a point of view, “spontaneous matheracy” compares favourably with “learned matheracy” and it is generally held that, far from being the privilege of a superior culture and the prerogative of academic *culchah,* mathematics is all around us, although in the form of *hidden* mathematics.

Nobody seems to have gone as far in that direction as Paulus Gerdes in his paper on culture, geometrical thinking and mathematics education in Mozambique (pp. 137-162). The article is made up of two very different parts. Whatever the political situation in Mozambique, let it be said that the first part (pp. 137-141) is a piece of sheer propaganda. It is certainly unusual to find, in a scientific paper, the unstinted encomium of a national hero. But it is especially regrettable to discover statements that so obviously and so deliberately fly in the face of facts, such as the following assertion (p. 140): “mathematics” – so writes Gerdes – “does not come from outside our African, Asian and American-Indian cultures”.

However, the paper’s main theses would deserve careful examination. The crux of the matter seems to lie in the following pronouncement (that Gerdes, following Nebres, fathers on Jacobsen): “The (African) people that are building the houses are not using mathematics; they’re *doing* it traditionally… if we can bring out the scientific structure of why it’s done, then you can teach science that way”. Gerdes improves on the story (p. 140): “In order to be able to incorporate [into the curriculum] popular (mathematical) practices, it is first of all necessary to *recognize their mathematical character*… A related problem is how to reconstruct mathematical traditions, when probably many of them have been – as a consequence of slavery, colonialism… – wiped out”. While the latter approach seems to remain currently out of reach of research, another method can be resorted to, that of “defreezing” so-called frozen mathematics (pp. 140-141): “The artisan, who imitates a known production technique, is, generally, not doing mathematics. But the artisan(s) who discovered the technique, *did* mathematics, was/were thinking mathematically. When pupils are stimulated to *reinvent* such a production technique, they are doing and learning mathematics”.

All this sounds like a (now more and more widespread) *private myth,* a wish-fulfilling daydream, a reconstruction of history that brazenly wanders from historical truth, to which at least three main objections can be made. Firstly, any social practice is subject to the objective laws of Nature: thus any product of human technology will naturally abide by the laws of geometry, of physics, etc. To use these laws, to rely on them as we do in everyday life does not amount to recognising them. I can pile up books or building-blocks on my writing-table, ride a bicycle or fly a kite without any knowledge of mechanics. Products of human activity are certainly amenable to mathematical modelling, but we have no reason to believe that the mathematics that one will eventually discover “in them” have once been consciously put into them. Secondly, there is no denying that in any culture there are traces of what might have developed into mathematics, physics, chemistry, etc. – traces that we may choose to call protomathematics, protophysics, etc. But only very few cultures have gone any way towards developing fully-fledged sciences. One simply should not mistake simmering water for water-vapour, nor, by ignoring the essential rôle of discontinuities in the history of science, indulge in a teleological view of that history. Thirdly, the striking thing about the whole story is the obstinate search for supposedly native mathematics, as if the presence or absence of mathematics in a given culture were a matter of life and death: as if it were *the* standard by which a culture should be judged. It certainly takes a mathematics noospherian to endorse such an ivory-towerish view, which most “aborigines” from all over the globe (including the writer) would bitterly resent.

CULTURAL PROBLEMS ARE COLLECTIVE ISSUES

The working model I earlier mentioned presents yet other blind spots. In drawing attention to the values purported to pervade mathematical activity, it also distracts attention from those values that mathematics *education*, not mathematics in itself, actively and often voluntarily conveys: values that, generally, cannot be imputed to mathematics as such. To quote a case in point, it is now commonly taken for granted that “awareness” is good and should be one major goal of education. But any *ignoramus* knows by experience that *some* degree of unawareness often helps – and this is true even in mathematics. (Mathematics is a perfect example on which a *celebration of ambiguity* could be founded.) To take another example, the history of mankind can be seen either as the easy-going history of will-power and *motivation* – a highly-praised value, but a prerogative of the victors – or, more realistically, as a tale full of sound and fury, as the history of submission and, at best, of *resilience and fortitude.* These latter values are what the average student and the working mathematician ordinarily need most.

Other papers in the volume under examination take a smoother approach to culture and mathematics education – one which seeks to take into account, and to rely on, what the child “brings to school”. But they do not fail, however implicitly, to pay tribute to the ingrained values of the (international) mathematics education community. In the article that opens the volume, Beth Graham deals with issues surrounding mathematics education and (Australian) aboriginal children, and offers a wide-ranging, noteworthy review of the relevant literature (pp. 119-135). In line with other experts’ conclusions, notably Bishop’s, emphasis is laid – very appositly, in my view – on the “critical rôle of language” as the basic instrument by which implicitly conflicting views can be made explicit and “talked through”, thus ensuring construction and negotiation of meaning and appreciation of significance: “concepts”, Graham asserts (p. 127), “can be ‘talked around’ in the everyday language of life”. The author relevantly goes on to show how much everyday language can be at a loss for words in the face of unprecedented, uncharted situations – a fact common to all languages. Lack of appropriate words, she remarks, can be compensated for by timely “language engineering”, i.e. by extending meanings, borrowing words or creating new words by combining two or three words (as in “right-angled triangle”). This entails a dynamic view of culture that happily departs from the more usual, static views.

Certainly the author is at her best whenever she draws on her experience of fieldwork. But the general idea advocated here remains subtly imbued with the typically Western – and “noospheric” – value of individualism: issues are raised, and solutions looked for, at the *level of the individual*, as if it fell upon each and every child to find the right way out. Cultural problems, however, are fundamentally *collective* problems, which befall social groups as such. Their solutions in individual cases are generally highly dependent on the attitude of the individual’s community, regarded as a community of interests both social and cultural. Accordingly, negotiations in the classroom should be conducted against the background of an overall *cultural negotiation*, without which every single individual solution will prove shaky, if not the exception that proves the rule.

The main issue in this respect merely boils down to the following: *what price, in terms of cultural tribute, is the community willing to pay, and for what advantages?* It is no wonder that some social groups will refuse, obstinately and knowingly, to overpay cultural goods that they might otherwise wish to secure for the younger generations. “Aboriginal people, Graham observes (p. 130), have been happy to have their children begin to be mathematical people…, for example, encouraging children to recognize and represent through drawing and language, the people that belong to a certain kin group. However, they may not be happy if the kinship system is dealt with in school in such a way that it becomes an ‘open’ system in Horton’s... sense of the word. To use it to encourage children to infer, predict, generalize and so forth may be considered inappropriate.” “Western education” then appears as a dearly bought advantage. The love story comes to a bad end. Cultural sensitivity on the part of the mathematics educator suddenly verges on illegitimate cultural inquisitiveness. The road to hell is paved with good intentions: where it does not sound like childish petting, the interest taken in the pupil’s culture often comes to resemble the predator’s interest in its prey. “By the time adults realized what was happening,” Graham continues, “it could already be too late.” The community may want their children to learn mathematics; they are certainly less enthusiastic about having them learn how to “explain mathematically” their traditional earthenware, or the geometrical and physical reasons which make the potter’s secrets and tricks really work. “We want them to learn English, an old Aboriginal man commented. Not the kind of English you teach them in class but your secret English. We don’t understand that English but you do.” Great expectations result in bitter disillusionment. “... after many years working in the Aboriginal context,” Graham concludes, “I now say ‘Take care’.”

CULTURE AS A SCREEN FACTOR

Norma C. Presmeg has contributed a paper on school mathematics in culture-conflict situations, that is really a study of the South African situation (pp. 163-177). The basic issue seems to be, how can one manage to teach mathematics in the current situation of social and political unrest? Strictly speaking, the same question could be put with respect to any other public utility – e.g., how can garbage collection continue to be ensured? Not unexpectedly, the study bears surreptitiously apologetical overtones. While the author "does not regard this paper as providing ultimate solutions to problems which are extremely complex" (p. 163), she claims that "even mathematics curricula… have a role to play in fostering mutual understanding amongst members of different cultures" – a conclusion that would hardly be applied to garbage collection. Once again, as is common usage in mathematics education literature, mathematics teaching is supposed to show the way to earthly salvation.

The political situation – the roots of which are known to everyone – is not ignored. Presmeg opens her article with a vivid, telling description of the effects of oppression and violence on the campus (pp. 163-165). But she readily focuses on cultural aspects, laying stress, for instance, on the recent transition from the slogan "Liberation first, education later!" to the more educationally promising catchword, "People's education for people's power!". The broad prospect of the negotiation mentioned previously – at the same time social, political and cultural – gives way to the more technical and circumscribed issue of "curriculum-development in a multicultural context". In this perspective, the mere recommendation, supported by many authorities, is put forward that the curriculum should be designed by a group of people "representative" of the diverse cultural groups involved – nothing being said, for instance, of the criteria according to which the choice of those "representatives" might be made.

Obviously, to confine oneself to cultural issues is not in itself scientifically illegitimate. The main problem is with the "robustness" of the conclusions reached. In the case in point, the domain of validity of any alleged "model for cultural change" should be carefully established, making it possible to check whether its hypotheses remain realistic under the prevailing conditions. With this in mind, one can appreciate the full import of Dr. Presmeg's considerations. Essentially she deems it possible for different cultures, which history has brought into contact, to come to mutual understanding through a commonly shared acculturation process. This optimistic view evokes a happy medium, far from both cultural fragmentation and cultural monism. The idea that cultural discontinuity – "living in two worlds" – is not inevitable, that an individual's relation to the world is not a one-piece thing, that it can make room for a peaceful diversity of cultural experiences, is forcefully expressed and exemplified, and a kind of cultural pluralism is more or less overtly encouraged. The importance of the stability of the cultural heritage and of its availability to the rising generation are emphasised and, conversely, in the wake of Margaret Mead, the rôle of "prefigurative enculturation" – adults learning from their children – is underlined. The rôle ascribed to schooling and the curriculum is seen as central to the "melting pot" experience advocated. The attention given to language as both an obstacle and an instrument, while following well-beaten tracks, develops into a doctrine whose tenets will not be repeated here. (Little is said, however, about the specifie part assigned to *mathematics* education in the overall process.) All that is well and good; one can, nevertheless, doubt whether what could succeed, for instance, for Germanic immigrants in Schönhausen (Federal Republic of Germany), in an area that gives "every appearance of social and economic health" (p. 168), will be of much value wherever human rights and the law of nations are so arrogantly disregarded.

FROM MATHEMATICAL PAROCHIALISMS TO 'UNIVERSAL' MATHEMATICS OR HOW TO BE A MATHEMATICAL ALIEN

Alan J. Bishop has condensed into a brief article his "fifteen or so years" of work on mathematics education in its cultural context (pp. 179-191). His is a composed, serene paper, yet both secretly passionate and slightly dubitative. More overtly than any of the previously examined contributions, it displays the pure logic that seems to have led to the already criticised views on mathematics in different, especially non-Western, cultures. The starting point of the whole story is certainly to be found in what is undeniably an established fact: in many countries, "like Papua New Guinea, Mozambique and Iran, there is criticism of the 'colonial' or 'Western' educational experience, and a desire to create instead an education which is in tune with the 'home' culture of the society" (p. 179).

What the form and content of such an education should be, certainly remains a real problem – and a difficult one. To say the least, however, the solution sketched here is highly debatable, if only because of its ambiguity. Firstly, a "cultural interface" of a sort is called for – a sensible demand in its own right. Secondly, it is convincingly argued that, whatever the culture (and, let us add, whatever the social milieu within a given culture), the child is likely to come into contact with a whole gamut of activities involving (proto)mathematical experiences, ranging from counting and measuring to "explaining" phenomena – a looser but obviously crucial category. All of these activities, let us then remark, are usually deeply contextualised, embodied as it were in definite, *culturally specific*, situations. Consequently, in order to rely on them one will have to attack the problem of their decontextualisation, and further recontextualisation, within the setting of *school* education. Whatever the difficulties, this is a sensible scheme, one on which, I would claim, traditional school education genuinely, if only partially, draws. Now, to go further in this direction would require patient "anthropological" analyses of the social practices accessible to the child, and still more *didactic* efforts to make the best of them in the classroom.

Such is not however the direction taken here. The anthropological and didactic problems that confront us at this point are swiftly lost from sight. Didactic considerations are made to stand aside and make way for an ambitious historical epistemology of mathematics. Referring to the six broad types of activity he has identified, Bishop writes (p. 183): "Mathematics, as cultural knowledge, derives from humans engaging in these six universal activities in a sustained, and conscious manner".[5] He then proceeds to supply a list of (mathematical) notions specifically arising from each of the six "universal activities" and comes to the conclusion that "From these basic notions, the rest of 'Western' mathematical knowledge can be derived, while in this structure can also be located the evidence of the 'other mathematics' developed by other cultures" (p. 184).

This is certainly a very appealing and seductively-presented theory of the historical genesis of mathematics, an attempt to account for its alleged polygenic development. However it remains open to much criticism from both didactic and historical points of view. On the one hand, if it is apt to provide the learner with "cultural" confidence and motivation, it is of little or no help in solving the main problems that the mathematics educator must face. On the other hand, as a historical epistemology of mathematics, it can be seriously questioned. For the facts of the case do not lend themselves easily to such an interpretation. While the polygenesis of *proto*mathematics seems beyond doubt, the inception of *mathematics* as such took place in the history of mankind under very special conditions. Mathematics certainly "took off" from protomathematics, but its emergence required more than mere

[5] The six "fundamental activities" which, Bishop claims, are "both universal… and also necessary and sufficient for the development of mathematical knowledge", are Counting, Locating, Measuring, Designing, Playing, and Explaining (pp. 182-183).

“consciousness”. At some point in the history of the world, for unknown reasons, people came to take a *reflexive* – not only conscious – view of what can now be thought of as *proto*mathematics. They even drew a sharp distinction between the know-how on which they began to ponder and the outcome of their speculation. This might well not have happened at all. In like manner, the Greek mathematical heritage could have been lost to mankind through default of heirs. The Romans proved to be poor mathematicians. Centuries later, the legacy fell to the Arabs, a civilisation without whose efforts and mathematical genius we, mathematics educators, would most certainly not exist today. An untutored ‘Western’ world – then, and for centuries to come, a world of peasants – hesitatingly took over. Many times the waters of history could have closed over that new Atlantis, mankind’s mathematical treasure. There was no easy way from the protomathematics of Babylon and Egypt to the mathematics of the Greeks, nor from the Greeks to present-day ‘Western’ mathematics.

*Proto*mathematics was always a sure thing, a very probable outcome of human activity. But *mathematics* has been a highly improbable venture. Mathematical experience always proved at variance with the ambient culture, and its history is full of forgotten cultural turmoil and discord. Actually, it is very unlikely that mathematics will ever be fully consonant with *any* culture whatsoever. Leaning on L. A. White’s *The Evolution of Culture* (1959), and regarding mathematics as a cultural phenomenon, Bishop provides a “wide-meshed” analysis of the values that, in his view, are carried by ‘Western’ mathematics. The axiology of mathematics activity is described as being made of pairs of corresponding opposites, *progress* and *control*, *rationalism* and *objectivism*, *openness* and *mystery*, all of which are explained *con brio* (pp. 184-187). “These then, Bishop sums up (p. 187), are the three pairs of values relating to Western mathematics which are shaped by, and also have helped to shape, a particular set of symbolic conceptual structures. Together with those structures they constitute the cultural phenomenon which is often labelled as ‘Western Mathematics’.”

Distinguishing between enculturation – “inducting the young into part of their culture” (p. 187) – and acculturation – inducting “the person into a culture which is in some sense alien” – Bishop gives a fair, well-balanced account of the complexity of the issue that the distinction implicitly raises. Considering ‘Western’ mathematics, he wonders “for which children is enculturation the appropriate model? Is [‘Western’ mathematics] really part of anyone’s *home* and *local* culture?” He then goes on to raise more questions on the relevance of thinking in terms of enculturation, and prudently comes to the conclusion that “different societies are influenced to different degrees by this international mathematico-technological culture”, and that “the greater the degree of influence the more appropriate would be the idea of enculturation” (p. 188). While the case of enculturation is thus wisely left undecided, acculturation is faced up to. Bishop balks at the idea of *intentional* acculturation, and accepts acculturation as “a natural kind of development when two cultures meet” only to conclude: “It might be possible to develop a bi-cultural strategy, but that should not be for ‘aliens’ like me to decide”.

The author seems to dream of peaceful encounters between cultures and pins his hopes on the emergence of a “culturally-fair” mathematics curriculum, “a curriculum which would allow all cultural groups to involve their own mathematical ideas whilst also permitting the ‘international’ mathematical ideas to be developed”. In this line of argument, he suggests that to start with the six universal activities already mentioned “would allow the mathematical ideas from different cultural groups to be introduced sensibly” (p. 189). Whereas this is only a sketch of a solution – a sensible one indeed – it is regrettable that, in his conclusion, he leaves the teacher to bear the brunt of the effort: “Teacher education”, Bishop asserts (p. 190), “is the key to cultural preservation and development”. I would suggest that this is too often an unfair, overworked trick, slyly resorted to to conceal our own ignorance, and our inability to find genuine solutions.

Like most other papers in the volume, Bishop's article abounds in comments and observations that are well worth meditating. Such is the distinction between "mathematics *training*" and "mathematics *education*", or that between "teaching values" and "teaching about values", for instance.[6] All these elements, unfortunately, add up to a rather fragile whole, permeated with moral sentiments. Thus progress, rationalism and openness are good, while control, objectivism and mystery are bad (see p. 189).

One cannot but wonder at the general atmosphere of unassertive uncertainty that pervades almost all statements and leads to guilt-ridden rationalisations. As a collective historical debt of the colonial powers, the guilt is beyond doubt. Its rationalisation is ineffective and may be grossly deceptive. It leads to the fascination for cultural genuineness – a very slippery concept in itself – and the propensity for looking back on a mythical past instead of looking forward to future developments. Mathematics is certainly not independent of culture and society. All societies on which it was grafted in the course of its history have contributed to shape and enrich this common good of mankind. Each of them has left its mark and, by its cultural style, its often specific centers of interest, its forms of social and scientific organisation, has set its own imprint on the development of mathematics. However, if I refer to the "rich Hungarian mathematical tradition", to take but one example, none of us will think of the Magyars' protomathematics as a decisive factor; and, at the same time, no one will deny the influence that "Hungarian mathematics" have exercised, for the benefit of all, on "universal mathematics". For the universality of mathematics is the "inductive limit" of successful and undaunted mathematical parochialisms.

THE 'OLD' MATHEMATICAL WORLD AND EMERGING 'MICROCULTURES'

Colonialism is silently – and devastatingly – taking its toll on Western consciences. This moral and cultural, not yet fully economic, backlash will not be felt in the study that Richard Noss devotes to the computer as a cultural influence in mathematical learning (pp. 251-268). Following Mellin-Olsen, he remarks from the start (p. 251) that "many if not most of the children in Western classrooms, are confronted with the mathematics of a subculture of which they are not – and perhaps have no wish to be – members". The general idea is therefore to bridge the gap between the learner's home culture and the culture afforded by the school environment, by supplying computer-based learning environments endowed with (in Papert's words) increased "cultural resonance".

The paper is carefully structured. The first part (pp. 252-254) raises the question, what does it mean to do mathematics? At first the inquiry trifles with the authorised dictum that all children naturally engage in mathematical activities in the context of everyday life, and that, although unconsciously, we are all mathematicians. (Noss even calls on Gramsci's help to put a polish on this rather insubstantial notion.) But this readily turns into an alert criticism of "those who have considered the importance of culturally embedded mathematical activity" (pp. 252-253). Taking up arguments developed by Keitel and Hoyles, Noss reduces the embedded mathematical structures to be merely a possible starting point for doing mathematics, and promptly raises the crucial issue: which tools are necessary to turn the contemplation of visible regularities (as apparent, for instance, in basket-making) into convincing mathematical activity? "What we have not had at our disposal", he observes (p. 254), "was the means for learners to engage in culturally embedded activities while simultaneously mathematising their activities". The need for abstraction and formalisation

[6] The distinction between "scientific training" and "scientific education" appears in a short piece on the teaching of the history of science, which Imre Lakatos wrote about 1962 (see Lakatos, 1978, pp. 254-255). It was certainly intended as a polemical weapon suited to draw a tactically meaningful demarcation line. One may wonder whether Lakatos would have regarded it as an honest concept of epistemology.

"essentially algebraic in character", its centrality to the "reflection on and synthesis of the mathematical relationships embedded within the activity" (p. 253), are excellently emphasised, and the method of learning through defreezing hidden mathematics is criticised for being a haphazard, unsystematic, and therefore didactically unreliable *modus operandi*.

The author then tackles the problem of the complex relationship between technology and culture (pp. 254-255). He draws a parallel between Third and First world countries, and comes to the (deceptively) paradoxical conclusion that, although the latter obviously embody much more mathematics (through the use of technology), such mathematical elements remain generally culturally invisible. "How much of the science and history of human culture is frozen into the production of a single sheet of paper?", he wonders (p. 257).[7] That science and history are therefore not readily available to the teaching/learning process. The cultural secretion of socially effective mathematics makes it a central problem to develop appropriate learning environments, as the case of algebra, for instance, clearly shows.

Noss next tries to come to terms with the racking question of the (apparent) meaninglessness, and related "dehumanising", of much of school mathematics, as exemplified in the so-called whimsical or pseudo-realistic problems. He does not seem to be aware that the stereotyped disguises of most traditional – and, for that matter, classical – problems are not specific to school mathematics. In fact, in the absence of an appropriate formalism – it is the lot of mathematicians to be always in search of adapted formalisms and to be often reduced to using provisional ones – those hackneyed formulations so readily anathematised, which tell us about sweets and toffees (or pipes and bricklayers), provide the building-blocks of *classical standard mathematical models*. They are not in themselves "real-life" situations. They are only more or less realistic *models*, i.e. more or less relevantly simplified representations of *real-life situations*. A situation involving "Christians throwing Turks overboard", to take up Noss' exampIe (pp. 255-256), can perfectly well be *modelled* in terms of beads and other familiar artefacts. The main problem to be carefully dealt with in going from reality to model, i.e. from "concrete nonsense" to "abstract nonsense" (as N. Steenrod once called general category theory), is that of jettisoning those features of the situation that are (or seem to be) immaterial to the problem. In other words, one needs to "skim the fat off" the situation, in order to get the pith of it. This is exactly what "mathematising" means, and even though, thanks to the language of (linear) algebra, they are nowadays almost always dispensable and most often discarded, the oft-told anecdotes of classical mathematics have deserved well of mathematics. They can still be of help today, whenever a more germane, formal jargon is wanting.

One might feel uneasy about a mathematics education that, under the influence of fashionable notions, has lost sight of these essentials of mathematical culture. Moreover, in arguing, as Noss does, that computers can put some engaging flesh on the not very attractive skeleton of school mathematics – a point that will not be disputed here – one also runs the risk of forgetting that computers are very noisy little critters; that consequently, to get to the core of the situation, the student will have to make his way through all that noise – a parasitic signal liable to interfere with the "didactic signal" intended for him (provided that such a signal, i.e. that a didactic intention, really exists). This is precisely the problem that, by reducing the number of potential distractors, the traditional use of stereotyped, pseudo-realistic formulations was suited to solve. This is also one of the many pitfalls that the didactic exploitation of the computer may bring back to the fore.

Understandably the author looks on the bright side of things. The computer, he observes (p. 257), is something that children see as "theirs" and is definitively part of their own subculture, much as Elvis was part of the author's – and, for that matter, of the writer's – subculture. (Without wishing to cavil, might it not be a short-lived rejoicing, not entirely free

[7] The problem is broached in Chevallard (1988).

from ambiguity? Should education help the child to retire into a shell of his own? Or should he be urged to fully participate in the world around him?) Be that as it may, it is in this general context that Noss then proceeds to appraise the full import of the computer with respect to mathematics education. The author, whose research work on Logo (mainly in association with Celia Hoyles) has attracted attention, is certainly not a narrow-minded Logo enthusiast. (Let us note here that he deliberately uses the word Logo in a broad, extensive sense, "as a placeholder for a certain kind of interaction with the computer": p. 258). It is beyond argument, he claims, that "by learning Logo, the child is behaving as a mathematician – is essentially doing mathematics". But – very relevantly – he raises the question of *what kind* of mathematics the student is most likely to come across in this way; of the extent to which "the mathematics of the computer culture intersect with the broader mathematical culture".

UNEXPECTED NOISE IN COMPUTER-LAND

The quest for an answer takes up the rest of the paper. Successively examined are the questions, what mathematics may the children *do*, *learn*, *be taught*? Relying on convergent research findings, Noss proposes that the computer should be regarded essentially as "enlarging the culture within which the child operates" (p. 260). More accurately, the computer is not only something that the student can get feedback from – a crucial aspect, but one often unduly emphasised. It also provides the student with appropriate tools to engage in effective mathematical activity, because it meets the "need for formalisation", which the author happily recognises as basic to mathematical experience and culture. "In proposing this explanation, Noss convincingly argues, I am emphasising the opportunity afforded by the Logo environment to use symbols in a meaningful context – to pose and solve problems with symbols rather than to play with 'concrete' situations which subsequently (and often artificially) require symbolisation".

Such a statement is a sign of the times. In the past decades, the mathematical noosphere – more accurately, the English-speaking mathematical noosphere and its cultural satellites – has been continually overwhelmed by the "outer world" culture and made to confess articles of faith such as the inconsequent assertion "mathematics is all around us", etc. Appropriately, Noss refers to the ongoing debate by drawing the reader's attention to a short, neat paper by David Pimm.[8] The noosphere seems now to regain self-assurance and to recover its faith in… mathematics. The small hours are over, it is almost dawn. At least some few unfetterred noospherians take it upon themselves to reassert the intrinsic *cultural discontinuity* between mathematical and everyday cultures. They challenge sanctified tenets of a once flourishing creed and rediscover the ancient wisdom that inspired Hadamard to say that concreteness is simply abstraction become familiar. Will they go so far as to appreciate that, to be at peace with itself, any "integrated" culture must make room for any number of such cultural discontinuities?

The answer to the question, "what mathematics *may* children learn?", is in keeping with this general orientation. Spontaneous, "Piagetian" learning does not provide the child with the opportunities of suitably changing his relation to such culture-laden notions as length and angle, for instance. A Logo-based learning environment appears to have definite effects in this respect. A group of 84 children who had studied Logo for one year and a group of 92 who had not, were compared on a set of paper-and-pencil tasks "designed to probe children's conceptions of length and angle" (p. 261). The match turns to the Logo children's advantage, especially where angles are concemed – a fact reasonably foreshadowed by Papert's own findings. I was, however, not entirely convinced by the – rather clumsy – argument (p. 262)

[8] See Pimm (1986).

about the higher achievement level of the Logo girls, which smacks of the “dormitive virtue” of opium (girls are said to achieve better because a Logo-programming environment is more appropriate to their “cognitive styles”). But it remains highly probable that some environments – including Logo-based environments – are more congenial to mathematical mores than many other more familiar ones.

So far, the Logo world is little more than a quarry from which material can be freely gathered, but whose treasures can also be altogether ignored. If it falls to the teacher to make good use of its potential, it is precisely at this point that research into (what I would call) *didactic engineering* could be most effective. Spontaneous learning situations thus give way to intentionally arranged *didactic* situations, in which the dialectic between problem and setting must be carefully organised. For, as Noss aptly remarks (in relation with, notably, Lave’s research findings on “everyday cognition”), “people and settings simultaneously *create problems* and *shape solutions*” (p. 263; my emphasis). Recent and ongoing research in this area, at the University of London Institute of Education and presumably elsewhere, is pointing to a promising future. The theorisation that the author cursorily outlines draws essentially on such notions as Vygotsky’s “zone of proximal development” and Bruner’s “scaffolding”, which both revolve round the idea of the right amount of collaboration that should be provided for the learner to enable him to master new concepts – a process in which the teacher is called on to play a crucial and “considerably more subtle” part. Much has been tried out in this respect. “Microworlds”, relating to definite subsets of the mathematics curriculum, have been created or are currently considered for study, for instance. However, it is obvious that hitherto no proof has been given that “computer culture” (in Papert’s words again) can be integrated in the mathematics classroom. Moreover, the intended change might prove something of a commotion to the didactic ecology of mathematical knowledge. Now, as is almost always the case with innovation, one is tempted to focus on the expected benefits of change, heedless of unintended consequences that might well destroy *latent* functions essential to the teacbing and learning process. The author, it is fair to say, never fails to maintain a wise reserve. But, in my opinion at least, caution is not enough. It is the moral duty of the researcher, whatever his personal beliefs and expectations, to stoop to look at the obverse truth. Needless to say, it is not at all unlikely that such a move will pay high dividends in terms of scientific results.

MATHEMATICS EDUCATION AS A SOCIAL INSTITUTION

It is precisely the purpose of Thomas S. Popkewitz – in a study headed “Institutional issues in the study of school mathematics: curriculum research” (pp. 221-249) – to seek to understand “the complex dynamics of pedagogical actions” (p. 222) in the line of “Durkheim’s observation at the turn of the century that he knew of no instance in which theories of change have gone into practice without great modification, and unintended and unwilled consequences”. As this quotation and other references show, the intellectual world of the author extends beyond the usual cultural limits of the noosphere, encroaching as it does on the fields of history, sociology and social psychology. At first sight, his contribution looks like a stray sheep in a gaggle of geese. In point of fact, it was first prepared as a report for the U.S. National Science Foundation (with a view to contributing to the establishment of a school mathematics monitoring center), and an earlier draft appeared in the journal of the Spanish Ministry of Education.

It is a comparatively long paper, carefully couched in unremitting, classical sociologese. It is also a wordy paper. But the wordiness is not accidental. It is almost self-explanatory. The world depicted in the ordinary literature on mathematics education is a simple, ungarnished world, a semi-vacuous space, a world in its pristine state. Popkewitz’s paper portrays a complex, labyrinthine, traumatic universe, teeming with beings and entities of all kinds, a world familiar and yet strange. The author opens up Pandora's box, obscures the orderly arrangement of the mathematics educator's privacy, threatens our intimate connexion with the once clear-cut universe of mathematics teaching. His emphasis is on the world around us *as it is*. He makes explicit what we usually take for granted and are no longer aware of. He implicitly challenges our *Weltanschauung*. He makes it obvious that our world view is too often that which the teaching *institution* imposes on us: a vision which, for the institution's sake, in order to preserve its smooth functioning, makes us oblivious of the institution as such.

The author first takes up the question, what social and cultural issues underlie the institutional patterns of schooling? The institution, Popkewitz argues, pervades every aspect of the teacher's and the learner's activity. It shapes – and confers meaning to – their decisions, behaviour, attitudes and emotions. It is the true arbiter of change. "Change", he writes consistently (p. 223), "requires an understanding of how the introduction of new practices interrelates with the existing structures of rules to challenge, modify or legitimate those arrangements". Mathematics education is not an island kingdom. It conveys, and embodies, and relies on – and is subjected to – much more than it purports to be. It is tied to its institutional setting and is therefore also riveted to society. Thus "the topic, the organization and the social messages all reflect assumptions about the nature of knowledge as defined within the confines of schooling which are not necessarily those of a mathematics discipline" (pp. 225-226).

Schooling makes a claim to homogeneity and generates differentiation. There are, Popkewitz reminds the reader (p. 226), "different forms of schooling for different people". These different forms of schooling, he adds, "emphasize different ways of considering ideas, contain different social values, and maintain different principles of legitimacy and forms of social control". Such differentiations are not only the product of social conflicts and opposing interests and world views; they also bear testimony to those dynamic elements in society that, to a certain extent, help to reshape society and schooling – as the U.S. civil rights movement of the 1960s and the feminist protest movement of the 1970s show. Schooling is deeply-rooted in society. Even though homogeneity is sought after, old social and cultural conflicts readily result in a definite drift from the intended curriculum.

The author then emphasises "the social predicament of schooling" and stresses the pressures exerted on schooling regarded as a means of solving a wide, too wide variety of societal problems. This objective predicament, he remarks, is "often obscured by traditions that give symbolic coherence and reasonableness to school practices" (p. 233). In the spirit of what Max Weber labelled "bureaucratic rationalism", reality-as-it-is gives way to streams of "administrative theories of behavioral objectives, criterion-referenced measures and competency testing", which all "give little reference to the institutional roles of schooling" (p. 234) and constitute so many rationalisations of reality that conceal "the roles of social relations and ways in which schooling articulates patterns of control and power". In so far as the teaching of mathematics is concerned, three main aspects, which "have little to do with conventional definitions of learning", emerge. Firstly, although taught mathematics "rarely, if ever, goes beyond 19th century mathematics", mathematics education is emblematic of a society that professes ideas of progress and enlightenment, rationality and scientific organisation. Secondly, education helps mathematics to stand out as a status symbol that commends itself to all, and whose recognition is forced even on those who "cannot master its codes". Thirdly, and no less important, mathematics education gives credit to an image of reality – described in terms of profits, budgets, and so forth – as an objective, transcending world that intrinsically stands beyond the reach of citizens and outwits individual agency.

Popkewitz then tackles the issue of "curriculum languages" (pp. 235-239). He thoroughly contrasts the mathematics of mathematicians – its inventivity, communality and openness – with the allegedly miserable condition of mathematics teaching. For the sake of the cause, it seems, the description is a little one-sided. The author expeditiously paints a gloomy picture of school mathematics. But he affects to see "scholarly" mathematics through rose-coloured spectacles. It turns out that this purple patch serves as a prelude to an all-out attack on the current U.S. educational "philosophy" (pp. 238-239). The main targets are individualism and "educational psychology", which, he asserts (p. 239), "involved the development of an academic discipline concerned with the successful adjustment of the individual to the environment". "The practical concerns of psychology, he continues, gave focus to a discourse about schooling which was functional in nature, and objective in method, and which transformed moral, ethical and cultural issues into problems of individual differences". I can find no pat answer to that.

The third question to which the author applies himself – "What notions of change illuminate the social complexities that inform the teaching of mathematics?" (p. 239) – leads him to call into question current – one could say *dominant* – analyses of curriculum change and educational research itself. Most research "results", he argues, are vitiated by a fundamental flaw, which methodological refinements are unlikely to repair. Almost all "models of change" actually involve unstated, clandestine hypotheses about the social world. Such assumptions as are traditionally relied upon can be reduced to three main types. There is first a general presupposition that the world conforms to the rationality of the model, or, more accurately, that the structure of the advised action on reality – the orderly, unilinear sequence of steps which usually make it up, mostly for administrative reasons – is isomorphic with its possible effects on the social world – as if the recipe could "explain" the cake. "The 'noise' of cultural and social interactions", Popkewitz observes (p. 242), "the complexities of causation that involve nonrandom practice and relational dimensions as part of the social order, and the role of human purpose are lost". In accordance with his view of social complexity, the author then goes on to question the implicit belief that desirable change occurs as the harmonious sum total of local actions held to be relevant – more computers, better trained teachers, and so forth. In this case, some kind of pre-established harmony is tacitly posited between the overall social functioning and man's wishes and intervention – an ungrounded postulate and one often belied by history. All change, it is mistakenly supposed, would be naturally adaptive, directional, irreversible and purposeful, as if guided by some benevolent and invisible hand. This (small-scale) "evolutionary" model of change is further contrasted with a third approach, which would happily bring together dialectics and change. The historical dynamics of social life clearly contradict naive expectations, and many obvious examples can be called on to "illustrate the complexities and unforeseen consequences of social action which must be attended to when considering issues of monitoring" (p. 244). All intended action, it is urged finally, should be analysed against the background of the totality of its social and historical "contexts".

On this and other points that the author makes, I will however not indulge in too much appreciative comment. Popkewitz's contribution no doubt provides food for thought. But most of the niceties to which he treats the reader have been the common stock of modern sociology since, at least, Max Weber. The paper is in many places reminiscent of C. Wright Mills' *The Sociological Imagination* (1959), a book whose illuminating lesson has not been widely learnt and is, at best, regularly unlearnt. If hopeless, it seems therefore not unreasonable to come back to it periodically. In the long run however, it becomes self-contradictory merely to assert again and again – because expected change, both in the research community and in the noosphere at large, fails to occur – a proclamation to the effect that desired change cannot prevail unless one considers the contexts in which it is bound to happen. Does it make sense any longer simply to hammer out words of which the addressees seem unable to make any sense at all? The right question, it seems, would be about the reasons why this is so: why "scientists", who once gave pride of place to intellectual craftsmanship – to use Mills' phrase – have consented to bow to readymade administrative theories; why they have gladly allowed themselves to be turned into "experts", and mandarins, and advisors to the King; why, in short, they have agreed to serve as foils to the polity's short-sightedness and illiberality. An ungarbled version of this story is still wanting.

THE PROBLEM OF DIDACTIC CONVERSION

On one other point at least Popkewitz's version is incomplete. Research on mathematics education must certainly broaden its outlook, and take into account determinants which it has so far flippantly ignored. It is also its duty, nevertheless, to investigate patiently, even punctiliously, the relationship between the individual's experience and conduct and the socially determined contexts in which they emerge. Now, by invoking unspecific social and cultural factors, one falls short of providing a satisfactory explanation of the pupil's (and the teacher's) behaviour. Just as tuberculosis cannot be explained by poverty and destitution, so one cannot account for the situations that happen in the classroom and elsewhere in terms of "didactic epidemiology" only. Sociological considerations offer clues that would of themselves only explain away what really remains to be construed in more specific terms. (Tuberculosis used to be connected with poverty, but it was, and *still* is, "explained" by the tubercle bacillus, independent of the prevailing economic conditions.)

The researcher has therefore to face the problem of *didactic conversion* and to discern the missing links. How, he should ask, and under what (extra) conditions, can such and such factor – let us say, such value, or such social status, or such culture norm, for instance – translate into precisely this or that behaviour? How can general conditions come to bear on, or materialise into, concretely observable behaviour patterns? How, in short, can "didactic etiology" confirm or disprove the evidence offered by looser epidemiological investigations?

It is precisely this kind of issue that the paper by *K. C. Cheung* on mathematics achievement and attitudes towards mathematics learning in Hong Kong (pp. 209-219) seems to tackle. Data from the Second IEA Mathematics Study are drawn upon to suggest that "the three attitude dimensions SELF, SOC and CREATE were the most pertinent dimensions in *explaining the variance* of mathematics achievement of Grade 7 students in Hong Kong" (p. 218; *italics added*). SELF "measures the students' own estimation of their ability in doing mathematics", SOC does the same for the "students' perception of the usefulness of mathematics in occupation and everyday life", and CREATE goes on to "measure the students' perception of the creativity in mathematics".

The study could have been a neat one. Unfortunately, it appears that the conclusion to which we jumped in the company of the author is a rickety one. To "explain the variance" is not to explain in any reasonable sense of the word. First, it only "explains" *the variance*, not the intrinsic reality itself; second, what it "explains" is explained in a technical sense whose relationship to a possible theoretical explanation remains to be established. It is a matter of common knowledge that statistical analysis of whatever kind can only grasp at correlations and that causal links elude it. The author is well aware of this and duly *declares* the relationship between attitudes and achievement to be, as a rule, "reciprocal" (see pp. 217-218). One might add that it would be equally reasonable, on strictly scientific grounds, to consider that achievement results in positive attitude change; that, to put it plainly, the student values mathematics simply *because* he or she is good at mathematics. The right answer – if there is any in such general terms, which one can doubt – could be only a theory-laden one. But Cheung seems to believe, from the outset, that this is only academic hair-splitting, to be acknowledged but not to be taken into further account. In fact, his conclusion turns out to be that "promoting the students' attitudes in these dimensions is likely to result in an increase in their achievement in mathematics in subsequent years of schooling" (p. 218). As could be expected, "explaining the variance" gives way to the straightforward statement of alleged “implications for teaching”.

Both the statistical analysis and the pedagogical hint may be beyond reproach. However, the gap between them is yet to be filled. That the flaw in the reasoning is not obvious to the author should also be explained. It may be that uncritical cultural insistence on the – supposedly crucial – rôle of motivation and expectations, on the one hand, the wish to come to a conclusion in terms of action to be taken, on the other, have been enough to obfuscate the problem. It should however be said, in the author’s defence, that this is what most “empirical research” in mathematics education currently boils down to.

THE NEED FOR MORE OPEN DISCUSSION

The book also includes four book reviews by distinguished colleagues. Especially worthy of note is the hard-hitting critique by D. D. Spalt. Some readers will perhaps consider it to be too harsh a reprimand, but it spurred the present writer to pungently express his views about the book as a whole. It is my belief, indeed, that, where mathematics education is concemed, the scientific debate that no research community can dispense with has gradually given way to mutual bowing and scraping, which leaves little or no room for healthy intellectual squabbling. Too often the noosphere seems to be an overprotected microcosm, where open discussion has given way to obsequious and back-scratching rituals within small circles. Our scientific democracy is haunted by the ghost of expertocracy; fields of research old and new are in permanent danger of being monopolised by a chosen few. The problems of mathematics education and culture should be everyone’s concem, and it is hoped, therefore, that many will choose to read this book, to come to grips with it, with a critical eye and an open mind. It is obviously worth the while.

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